

NEUTRINOLESS DOUBLE β -DECAY ¹

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Abstract

The neutrinoless double β -decay is reviewed. Model independent evidence in favor of neutrino masses and mixing is briefly summarized. The data of the recent experiments on the search for $0\nu\beta\beta$ -decay are presented and some future experiments are discussed. The possible values of the effective Majorana mass, which can be predicted on the basis of the neutrino oscillation data under different assumptions on the pattern of the neutrino mass spectrum, are considered. A possible model independent test of the nuclear matrix element calculations is discussed.

1 Introduction

The status of the problem of the neutrino mixing drastically changed during the last 5-6 years: in the atmospheric Super-Kamiokande experiment [1], the solar SNO experiment [2, 3] and the reactor KamLAND experiment [4] strong model independent evidence of neutrino oscillations was obtained.

There are many open problems of neutrino mixing. The value of the parameter $\sin^2 \theta_{13}$ is the most urgent one. The value of this parameter is crucial for the search of such fundamental effects of the tree-neutrino mixing as CP violation in the lepton sector. The value of the parameter $\sin^2 \theta_{13}$ will be measured in the future reactor [5] and long baseline accelerator experiments [6].

One of the most important problem of the neutrino mixing is the problem of the nature of the neutrinos with definite masses ν_i : are they truly neutral Majorana particles or Dirac particles, possessing conserved total lepton

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number? The solution of this problem will have a profound impact on the understanding of the origin of small neutrino masses and neutrino mixing.

The study of the neutrinoless double β -decay ($0\nu\beta\beta$ -decay)

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$$

is the most sensitive method of the investigation of the Majorana nature of neutrinos with definite mass. We will discuss here this process. In the next section model independent evidence of neutrino oscillations will be briefly reviewed. Then the data of the recent $0\nu\beta\beta$ -decay experiments and projects of the future experiments will be discussed. We will consider the possible values of the effective Majorana mass, which can be predicted from the results of the analysis of the neutrino oscillation data. In the last section we will discuss the problem of nuclear matrix elements of the $0\nu\beta\beta$ -decay. A possible test of the models of the nuclear matrix element calculations will be proposed.

2 Model independent evidence of neutrino oscillations

Compelling model independent evidence of neutrino masses and mixing was obtained in the atmospheric Super-Kamiokande [1], solar SNO [2, 3] and reactor KamLAND [4] experiments.

In the Super-Kamiokande (SK) experiment zenith angle θ_z dependence of the electron and muon atmospheric neutrino events was studied in details. If there are no neutrino oscillations the number of the high-energy neutrino-induced $\mu(e)$ events $N_{\mu,e}(\cos\theta_z)$ must satisfy the symmetry relation

$$N_{\mu,e}(\cos\theta_z) = N_{\mu,e}(-\cos\theta_z).$$

For the high-energy muon neutrinos this relation is strongly violated: the significant deficit of the up-going muons is observed. For the ratio of the total numbers of the up-going and down-going muon neutrinos the following value

$$\left(\frac{U}{D}\right)_\mu = 0.54 \pm 0.04 \pm 0.01 \quad (1)$$

was obtained. Here U is the total number of the up-going muons ($-1 \leq \cos\theta_z \leq -0.2$) and D is the total number of the down-going muons ($0.2 \leq \cos\theta_z \leq 1$).

The up-going muons are produced by neutrinos passing distances from about 500 km to about 13000 km and the down-going muons are produced by neutrinos traveling distances from about 20 km to about 500 km. The ratio (1) clearly demonstrates the dependence of the flux of the muon neutrinos on the distance between the neutrino production region in the atmosphere and neutrino detector.

The data of the SK experiment and other atmospheric neutrino experiments MACRO [7] and SOUDAN 2 [8] are described by the two-neutrino $\nu_\mu \rightarrow \nu_\tau$ oscillations. From the two-neutrino analysis of the SK data the following best-fit values of the oscillation parameters were found

$$(\Delta m^2)_{\text{SK}} = 2 \cdot 10^{-3} \text{eV}^2; (\sin^2 2\theta)_{\text{SK}} = 1.0 \quad (\chi^2_{\text{min}} = 170.8/170 \text{ d.o.f.}) \quad (2)$$

At the 90% CL the oscillation parameters are in the ranges

$$1.3 \cdot 10^{-3} \leq (\Delta m^2)_{\text{SK}} \leq 3.0 \cdot 10^{-3} \text{eV}^2; \quad (\sin^2 2\theta)_{\text{SK}} > 0.9 \quad (3)$$

In the SNO experiment [2, 3] solar neutrinos from the decay ${}^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu_e$ are detected via the observation of the CC reaction

$$\nu_e + d \rightarrow e^- + p + p \quad (4)$$

and the NC reaction

$$\nu_l + d \rightarrow \nu_l + n + p \quad (l = e, \mu, \tau) \quad (5)$$

For the total flux of ν_e detected via CC reaction (4) the following value

$$\Phi_{\nu_e}^{\text{SNO}} = (1.59_{-0.07}^{+0.09}(\text{stat.})_{-0.08}^{+0.06}(\text{syst.})) \cdot 10^6 \text{ cm}^{-2} \text{s}^{-1} \quad (6)$$

was obtained in the SNO experiment.

For the total flux of ν_e , ν_μ and ν_τ detected via NC reaction (5) the significantly larger value

$$\sum_{l=e,\mu,\tau} \Phi_{\nu_l}^{\text{SNO}} = (5.21 \pm 0.27 \pm 0.38) \cdot 10^6 \text{ cm}^{-2} \text{s}^{-1} \quad (7)$$

was found. Eq.(6) and Eq.(7) give us a model independent evidence of the transitions of the original solar ν_e into $\nu_{\mu,\tau}$.

The results of all solar neutrino experiments [2, 3, 9, 10, 11, 12] are described by the two-neutrino $\nu_e \rightarrow \nu_e$ survival probability in matter. From

the analysis of the data in the most preferable LMA region the following best-fit values of the oscillation parameters were found [2]

$$(\Delta m^2)_{\text{sol}} = 5 \cdot 10^{-5} \text{eV}^2; (\tan^2 \theta)_{\text{sol}} = 0.34; \chi^2_{\text{min}} = 57/72 \text{ d.o.f.} \quad (8)$$

In the KamLAND experiment [4] electron antineutrinos from many reactors in Japan and Korea are detected via observation of the process

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

The average distance between reactors and the detector in this experiment is 180 km. For the ratio of the total numbers of the observed and expected events the following value

$$\frac{N_{\text{obs}}}{N_{\text{exp}}} = 0.611 \pm 0.085 \pm 0.041$$

was obtained. This result is a clear evidence of the disappearance of $\bar{\nu}_e$'s on the way from the reactors to the detector.

The KamLAND data are described by the two-neutrino $\bar{\nu}_e \rightarrow \bar{\nu}_e$ oscillations in vacuum. The best-fit values of the oscillation parameters which were found from the analysis of the data

$$(\Delta m^2)_{\text{KL}} = 6.9 \cdot 10^{-5} \text{eV}^2; (\sin^2 2\theta)_{\text{KL}} = 1. \quad (9)$$

are compatible with the solar LMA values.

From common analysis of the solar and KamLAND data (assuming CPT) the following best-fit values of the parameters

$$(\Delta m^2)_{\text{sol+KL}} = 7.1 \cdot 10^{-5} \text{eV}^2; (\tan^2 \theta)_{\text{sol+KL}} = 0.41 \quad (10)$$

were obtained [3].

In [13] from the global two-neutrino analysis of the solar and the KamLAND data the following 90% CL ranges of the parameters were found

$$6.0 \cdot 10^{-5} \leq (\Delta m^2)_{\text{sol+KL}} \leq 8.7 \cdot 10^{-5} \text{eV}^2; 0.25 \leq (\sin^2 \theta)_{\text{sol+KL}} \leq 0.37. \quad (11)$$

SK atmospheric neutrino evidence of neutrino oscillations was confirmed by the long baseline accelerator K2K experiment [14]. The distance between neutrino source (KEK accelerator) and SK detector is about 250 km. Average ν_μ energy is 1.3 GeV. In the K2K experiment 56 ν_μ events were observed.

The expected number of the events is equal to $80.1^{+6.2}_{-5.4}$. The best-fit values of the two oscillation parameters found from the two-neutrino analysis of the data

$$(\sin^2 2\theta)_{\text{K2K}} = 1; \quad (\Delta m^2)_{\text{K2K}} = 2.8 \cdot 10^{-3} \text{ eV}^2$$

are compatible with the atmospheric values (3).

The negative results of the CHOOZ [15] and Palo Verde [16] reactor experiments are very important for the neutrino mixing. In these experiments reactor $\bar{\nu}_e$'s are detected via the observation of the classical process $\bar{\nu}_e + p \rightarrow e^+ + n$. The distances between reactors and detectors in these experiments were about 1 km. No disappearance of $\bar{\nu}_e$'s were observed. In the CHOOZ experiment for the ratio of the total numbers of observed and expected events the following value

$$\frac{N_{\text{obs}}}{N_{\text{exp}}} = 1.01 \pm 2.4\% \pm 2.7\%$$

was found. From the CHOOZ two-neutrino exclusion curve at $\Delta m^2 = 2 \cdot 10^{-3} \text{ eV}^2$ (the SK best-fit value) the following upper bound

$$(\sin^2 2\theta)_{\text{CZ}} \leq 2 \cdot 10^{-1} \quad (12)$$

can be obtained.

3 Neutrino oscillations in the framework of three-neutrino mixing

Neutrino oscillation data are analyzed under three basic assumptions (see, for example, [17]):

- Three flavor neutrinos ν_e, ν_μ, ν_τ (and antineutrinos $\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$) exist in nature.
- The Lagrangian of the interaction of the flavor neutrinos is given by the Standard Model. The lepton charged current and neutrino neutral current have the form

$$j_\alpha^{\text{CC}} = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_\alpha l_L; \quad j_\alpha^{\text{NC}} = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_\alpha \nu_{lL}. \quad (13)$$

- Neutrino mixing

$$\nu_{lL} = \sum_{i=1}^3 U_{li} \nu_{iL} \quad (14)$$

takes place. Here ν_i is the field on neutrino with mass m_i and U is the unitary PMNS [18, 19] mixing matrix.

The three-neutrino scheme (14) is the minimal scheme of neutrino mixing: the number of the massive neutrinos ν_i is equal the the number of the flavor neutrinos ν_l (three). If the number of light neutrinos ν_i is larger than three, sterile neutrinos must exist. An indication in favor of neutrino oscillations with the third relatively large neutrino mass-squared difference ($\simeq 1 \text{ eV}^2$) was obtained in the single accelerator LSND experiment [20]. The LSND result needs confirmation. It will be checked by the MiniBooNE experiment at the Fermilab [21] .

Neutrino oscillation data are compatible with two types of neutrino mass spectra².

1. Normal mass spectrum (NS):

$$m_1 < m_2 < m_3; \quad \Delta m_{21}^2 \simeq \Delta m_{\text{sol-KL}}^2; \quad \Delta m_{32}^2 \simeq \Delta m_{\text{SK}}^2.$$

2. Inverted mass spectrum (IS):

$$m_3 < m_1 < m_2; \quad \Delta m_{21}^2 \simeq \Delta m_{\text{sol-KL}}^2; \quad \Delta m_{31}^2 \simeq -\Delta m_{\text{SK}}^2$$

Here Δm_{SK}^2 and $\Delta m_{\text{sol-KL}}^2$ are neutrino mass-squared differences, which are determined from the two-neutrino analysis of the atmospheric SK and the solar-KamLAND data (see (2), (3) , (10) and (11)).

From neutrino oscillation data it follows that two independent neutrino mass-squared differences satisfy the inequality:

$$\Delta m_{21}^2 \ll \Delta m_{32}^2. \quad (15)$$

In the framework of the three-neutrino mixing we have (see [17])

$$4 |U_{e3}|^2 (1 - |U_{e3}|^2) = (\sin^2 2\theta)_{\text{CZ}}. \quad (16)$$

² $\Delta m_{ki}^2 = m_k^2 - m_i^2$. Notice that different notations for neutrino masses are used for NS and IS spectra

If we take into account solar data, from (12) and (16) we obtain that

$$|U_{e3}|^2 \leq 5 \cdot 10^{-2} \quad (17)$$

The present-day picture of neutrino oscillations is determined by the inequalities (15) and (17) (see, for example, [17]). In the leading approximation neutrino oscillations in the experiments with $L/E \simeq 10^3$ (L is the source-detector distance in m (km), E is the neutrino energy in MeV (GeV)) are two-neutrino $\nu_\mu \rightarrow \nu_\tau$ oscillations. Solar neutrino transitions in matter (and antineutrino oscillations in reactor experiments with $L/E \simeq 10^5$) are $\nu_e \rightarrow \nu_{\mu,\tau}$ ($\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$) oscillations. Solar ν_e and reactor $\bar{\nu}_e$ survival probabilities depends on two parameters Δm_{21}^2 and $\sin^2 \theta_{12}$ (if CPT is assumed) and are given by the standard two-neutrino expressions in matter and in vacuum, respectively.

The observation of neutrino oscillations means that the flavor lepton numbers L_e , L_μ and L_τ are not conserved by a neutrino mass term. There exist two theoretical possibilities in this case

1. The total lepton number $L = L_e + L_\mu + L_\tau$ is conserved. In this case fields of neutrinos with definite masses $\nu_i(x)$ are *Dirac fields* of neutrinos ($L=1$) and antineutrinos ($L=-1$).
2. The total lepton number L is violated. In this case the fields of neutrinos with definite masses $\nu_i(x)$ are *Majorana fields* of neutrinos (with antineutrinos identical to neutrinos). The fields $\nu_i(x)$ satisfy the Majorana condition

$$\nu_i^c(x) = C \bar{\nu}_i^T(x) = \nu_i(x), \quad (18)$$

where C is the charge conjugation matrix.

After the discovery of the neutrino mixing the problem of the nature of massive neutrinos (Dirac or Majorana?) is one of the most fundamental. There is no doubt that the solution of this problem will have very important impact on our understanding of the origin of small neutrino masses and neutrino mixing.

The investigation of the neutrino oscillations is an extremely sensitive method of the measurement of the very small neutrino mass-squared differences [22]. This method does not allow, however, to reveal the nature of massive neutrinos [23].

In fact mixing matrices in the Majorana and Dirac cases are connected by the relation

$$U^{Mj} = U^D S(\beta),$$

where $S_{ik}(\beta) = e^{i\beta_k} \delta_{ik}$ is a diagonal phase matrix. The matrix $S(\beta)$ does not give contribution to the transition probability

$$P(\nu_l \rightarrow \nu_{l'}) = \left| \sum_i U_{l'i} e^{-i\Delta m_{i1}^2 \frac{L}{2E}} U_{li}^* \right|^2.$$

Thus, we have

$$P^{Mj}(\nu_\alpha \rightarrow \nu_{\alpha'}) = P^D(\nu_\alpha \rightarrow \nu_{\alpha'}) \quad (19)$$

The nature of the massive neutrinos can be revealed only via the investigation of the processes in which the total lepton number L is not conserved. The most sensitive to small Majorana neutrino masses process is neutrinoless double β - decay of even-even nuclei (see reviews [24, 25, 26, 27, 28, 29, 30]):

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-.$$

In the case of the small Majorana neutrino masses the half-life of this process is given by the following general expression

$$\frac{1}{T_{1/2}^{0\nu}(A, Z)} = |m_{ee}|^2 |M^{0\nu}(A, Z)|^2 G^{0\nu}(E_0, Z). \quad (20)$$

Here $|m_{ee}|$ is the effective Majorana mass

$$|m_{ee}| = \left| \sum_i U_{ei}^2 m_i \right|, \quad (21)$$

$M^{0\nu}(A, Z)$ is nuclear matrix element, which depends only on nuclear properties, and $G^{0\nu}(E_0, Z)$ is known phase-space factor (E_0 is the energy release). In the next sections we will discuss

1. Existing data and future experiments.
2. Possible values of the effective Majorana mass $|m_{ee}|$.
3. The problem of nuclear matrix elements.

4 Existing $0\nu\beta\beta$ -decay data and future experiments

Neutrinoless double β decay is allowed for such even-even nuclei for which usual β decay is forbidden by the conservation of energy. There are several nuclei of this type:

$$^{76}\text{Ge}(2.039), ^{130}\text{Te}(2.528), ^{136}\text{Xe}(2.480), ^{100}\text{Mo}(3.034), ^{150}\text{Nd}(3.367)$$

and others. In the brackets the energy release E_0 in MeV is given. This is an important characteristic of the $0\nu\beta\beta$ -decay: the decay probability is proportional to E_0^5 .

The results of many experiments on the search for $0\nu\beta\beta$ -decay are available at present (see [31, 32, 33]). The most stringent lower bounds on the half-life of $0\nu\beta\beta$ -decay was reached in the Heidelberg-Moscow [34] and CUORICINO [35] experiments.

The detector (and source) of the Heidelberg-Moscow experiment [34] consists of 5 crystals of 86% enriched ^{76}Ge of the total mass about 11kg. For the half-life the lower bound

$$T_{1/2}^{0\nu} \geq 1.9 \cdot 10^{25} \text{ y} \quad (90\% \text{ CL}) \quad (22)$$

has been found. Taking into account different calculations of the nuclear matrix element, from (22) for the effective Majorana mass $|m_{ee}|$ the following upper bounds

$$|m_{ee}| \leq (0.3 - 1.2) \text{ eV} . \quad (23)$$

were obtained.

In the cryogenic experiment CUORICINO [35] TeO_2 crystals with a total mass 40.7 kg are employed. For the half-life of ^{130}Te in this experiment the following lower bound

$$T_{1/2}^{0\nu} \geq 7.5 \cdot 10^{23} \text{ years} \quad (24)$$

was reached recently. From (24) for the effective Majorana mass the upper bounds

$$|m_{ee}| \leq (0.3 - 1.7) \text{ eV} \quad (25)$$

were obtained.

Many projects of new experiments on the search for the neutrinoless double β -decay of different nuclei are under research and development at present (see [29, 32, 36]).

The main goal of the future experiments is to reach the sensitivity $|m_{ee}| \simeq$ a few 10^{-2} eV. This goal can be accomplished by detectors with mass about 1 ton or more, which have a good energy resolution, low background and an efficient signature for $0\nu\beta\beta$ events.

The experiment CUORE [35] will be a continuation of the CUORICINO experiment. Cryogenic detector will consist of 1000 TeO₂ crystals operated at a temperature 10 mK. The total mass of the detector will be about 800 kg. The expected resolution at $E_0 = 2.528$ MeV is 5 keV. For the half-life of ¹³⁰Te the value

$$T_{1/2}^{0\nu} \simeq 9.5 \cdot 10^{26} \text{ years} \quad (26)$$

is envisaged. This corresponds to the sensitivity

$$|m_{ee}| \simeq (2 - 5.2) \cdot 10^{-2} \text{ eV} \quad (27)$$

In the EXO experiment [32] up to 10 tons of 60-80 % enriched ¹³⁶Xe are planned to use. An important feature of this experiment is a laser tagging of Ba⁺ ions, produced in the recombination of Ba⁺⁺ ions from the decay ¹³⁶Xe \rightarrow ¹³⁶Ba⁺⁺ + e⁻ + e⁻. The detection of Ba⁺ ions will provide large background reduction. The value

$$T_{1/2}^{0\nu} \simeq 1 \cdot 10^{28} \text{ years} \quad (28)$$

is expected. It corresponds to the sensitivity

$$|m_{ee}| \simeq (1.3 - 3.7) \cdot 10^{-2} \text{ eV} \quad (29)$$

The GENIUS experiment [37] will be a development of the Heidelberg-Moscow experiment. About 1 ton of 86 % enriched ⁷⁶Ge will be embeaded in a large liquid nitrogen cryostat. The liquid nitrogen will provide effective shielding from the external background. For the half-life a value

$$T_{1/2}^{0\nu} \simeq 1 \cdot 10^{28} \text{ years} \quad (30)$$

is expected. This value corresponds to the sensitivity

$$|m_{ee}| \simeq (1.3 - 5.0) \cdot 10^{-2} \text{ eV} \quad (31)$$

In the MAJORANA experiment [36], which will be the continuation of the IGEX experiment [38], about 500 kg of 86 % enriched ⁷⁶Ge will be used. The main background is expected from the decay ⁶⁸Ge \rightarrow ⁶⁸Ga + e⁺ + ν_e .

It will be suppressed by the segmentation of the detector and effective pulse shape analysis of the signal. In the MAJORANA experiment the value

$$T_{1/2}^{0\nu} \simeq 4 \cdot 10^{27} \text{ years} \quad (32)$$

is expected. It corresponds to the sensitivity

$$|m_{ee}| \simeq (2.1 - 7.0) \cdot 10^{-2} \text{ eV}. \quad (33)$$

5 Effective Majorana mass

The effective Majorana mass is determined by the absolute values of the neutrino masses, mixing angles and CP phases. Let us discuss first these three ingredients.

1. Neutrino masses

From neutrino oscillation data only neutrino mass-squared differences can be inferred. In the case of the NS neutrino mass spectrum the neutrino masses are given by the relations

$$m_2 \simeq \sqrt{m_1^2 + \Delta m_{21}^2}; \quad m_3 \simeq \sqrt{m_1^2 + \Delta m_{32}^2}$$

For the IS neutrino mass spectrum we have

$$m_1 \simeq m_2 \simeq \sqrt{m_3^2 + |\Delta m_{31}^2|}$$

From existing data only upper bounds of the lightest mass $m_{\min}(m_1 \text{ or } m_3)$ can be obtained. From the data of the Troitsk [40] and Mainz [39] tritium β -decay experiments the following bounds were found

$$m_{\min} \leq 2.05 \text{ (2.3) eV} \quad (34)$$

More stringent bound

$$m_{\min} \leq 0.6 \text{ eV}$$

was obtained from the analysis [41] of the data of WMAP and Sloan Digital Sky Survey Collaborations.

2. Mixing angles.

The elements U_{ei} ($i=1,2,3$) are given by

$$U_{ei} = |U_{ei}| e^{i\alpha_i},$$

where α_i are Majorana CP-phases. If we introduce the angle θ_{13} in such a way that

$$|U_{e3}|^2 = \sin^2 \theta_{13}$$

from the unitarity relation $\sum_{i=1}^3 |U_{ei}|^2 = 1$ we have

$$|U_{e1}|^2 = \cos^2 \theta_{13} \cos^2 \theta_{12}; \quad |U_{e2}|^2 = \cos^2 \theta_{13} \sin^2 \theta_{12}.$$

The parameter $\sin^2 \theta_{12}$ can be determined from the results of the solar and KamLAND experiments. From the analysis of the existing data the best fit value (10) and a range (11) were found.

Only upper bound of the parameter $\sin^2 \theta_{13}$ is known today. From the exclusion curve obtained in the CHOOZ experiment the bound (17) was obtained.

3. Majorana phases

Majorana phases α_i are unknown. In the case of the CP invariance in the lepton sector the elements of the mixing matrix satisfies the relation [42]:

$$U_{ei} = U_{ei}^* \eta_i, \quad (35)$$

where $\eta_i = i \rho_i$ $\rho_i = \pm 1$ is the CP-parity of the Majorana neutrino ν_i . Thus, in the case of the CP invariance

$$U_{ei}^2 = i |U_{ei}|^2 \rho_i$$

and the effective Majorana mass takes the form

$$|m_{ee}| = \left| \sum_{i=1}^3 |U_{ei}|^2 \rho_i m_i \right| \quad (36)$$

From this equation it follows that in the case of the different CP parities of Majorana neutrinos with definite masses cancellations of their contributions to the effective Majorana mass can take place. In the general case of the CP violation $|m_{ee}|$ depends on two Majorana phase differences.

The value of the effective Majorana mass $|m_{ee}|$ strongly depends on the pattern of the neutrino mass spectrum and lightest neutrino mass [43]. Three types of the neutrino mass spectrum are usually considered.

- Neutrino mass hierarchy ³

$$m_1 \ll m_2 \ll m_3. \quad (37)$$

In the case of the neutrino mass hierarchy m_2 and m_3 are determined by the “solar-KamLAND” and “atmospheric” mass-squared differences

$$m_2 \simeq \sqrt{\Delta m_{21}^2}; \quad m_3 \simeq \sqrt{\Delta m_{32}^2} \quad (38)$$

and the lightest neutrino mass m_1 is small: $m_1 \ll \sqrt{\Delta m_{21}^2} \simeq 8.4 \cdot 10^{-3} \text{eV}$.

Neglecting the contribution of m_1 to the effective Majorana neutrino mass, we have

$$|m_{ee}| \simeq \left| \cos^2 \theta_{13} \sin^2 \theta_{12} \sqrt{\Delta m_{21}^2} + e^{i2\alpha_{32}} \sin^2 \theta_{13} \sqrt{\Delta m_{32}^2} \right|, \quad (39)$$

where $\alpha_{32} = \alpha_3 - \alpha_2$. It follows from (39) that $|m_{ee}|$ is small: the first term is small because of the smallness of $\sqrt{\Delta m_{21}^2}$; the contribution of the “large mass” $\sqrt{\Delta m_{32}^2}$ is suppressed by the smallness of the parameter $\sin^2 \theta_{13}$. From (3) (11) (17) and (39) we obtain the following upper bound (90 % CL):

$$|m_{ee}| \leq 6.0 \cdot 10^{-3} \text{eV} \quad (40)$$

The bound (40) is significantly smaller than the sensitivity of the future experiments on the search for $0\nu\beta\beta$ -decay. The observation of the neutrinoless double β -decay in the experiments of the next generation would mean that the neutrino masses do not follow the hierarchy (37).

- Inverted hierarchy of neutrino masses

$$m_3 \ll m_1 < m_2. \quad (41)$$

³Notice that masses of quarks (up and down) and charged leptons follow the hierarchy of the type (37).

In the case of the inverted mass hierarchy m_1 and m_2 are determined by “atmospheric” mass-squared difference

$$m_1 \simeq m_2 \simeq \sqrt{|\Delta m_{31}^2|} \quad (42)$$

and the lightest neutrino mass m_3 is small: $m_3 \ll \sqrt{|\Delta m_{31}^2|} \simeq 4.5 \cdot 10^{-2} \text{eV}$.

Neglecting small contributions of m_3 and $\sin^2 \theta_{13}$, for the effective Majorana mass we have

$$|m_{ee}| \simeq \sqrt{|\Delta m_{31}^2|} \left| \sum_{i=1,2} U_{ei}^2 \right| \simeq \sqrt{|\Delta m_{31}^2|} (1 - \sin^2 2\theta_{12} \sin^2 \alpha_{21})^{\frac{1}{2}}, \quad (43)$$

where $\alpha_{21} = \alpha_2 - \alpha_1$. In the case of the CP invariance $\alpha_{21} = \frac{\pi}{4}(\rho_2 - \rho_1) = 0, \pm \frac{\pi}{2}$.

From Eq.(43) for the effective Majorana mass we have the following range

$$\cos 2\theta_{12} \sqrt{|\Delta m_{31}^2|} \leq |m_{ee}| \leq \sqrt{|\Delta m_{31}^2|}, \quad (44)$$

where the upper and lower bounds correspond to the cases of the CP conservation: the upper bound corresponds to the case of equal CP parities of ν_1 and ν_2 and the lower bound corresponds to the case of opposite CP parities.

From (44) and (11) at 90% CL for the effective Majorana mass $|m_{ee}|$ we have the range

$$0.26 \sqrt{|\Delta m_{31}^2|} \leq |m_{ee}| \leq \sqrt{|\Delta m_{31}^2|} \quad (45)$$

Thus, in the case of the inverted hierarchy of neutrino masses the scale of the effective Majorana mass is determined by $\sqrt{|\Delta m_{31}^2|}$. From (45) and (3) we have

$$0.9 \cdot 10^{-2} \text{eV} \leq |m_{ee}| \leq 5.5 \cdot 10^{-2} \text{eV} \quad (46)$$

The values of the effective Majorana mass in the range (46) can be reached in $0\nu\beta\beta$ -decay experiments of the next generation. Let us stress

that in the case of the inverted hierarchy of the neutrino masses the value of $|m_{ee}|$ can not be smaller than $\simeq 1 \cdot 10^{-2} \text{eV}$. This is connected with the fact that the value $\sin^2 2\theta_{12} = 1$ (maximal 1-2 mixing) is excluded at 5.4σ level by the solar and KanLAND data [3].

From Eq. (43) we find

$$\sin^2 \alpha_{21} \simeq \left(1 - \frac{|m_{ee}|^2}{|\Delta m_{31}^2|}\right) \frac{1}{\sin^2 2\theta_{12}}. \quad (47)$$

If the problem of the nuclear matrix elements will be solved (see the next section for discussion), the experiments on the measurement of the half-life of $0\nu\beta\beta$ -decay could allow to obtain an information on the the value of the CP parameter $\sin^2 \alpha_{21}$ [44].

- Practically degenerate neutrino masses

If the lightest neutrino mass is much larger than $\sqrt{\Delta m_{32}^2} \simeq 4.5 \cdot 10^{-2} \text{eV}$ in this case neutrino masses are practically degenerate:

$$m_1 \simeq m_2 \simeq m_3. \quad (48)$$

For the degenerate neutrino masses the effective Majorana mass

$$|m_{ee}| \simeq m_0 \left| \sum_{i=1}^3 U_{ei}^2 \right| \simeq m_0 (1 - \sin^2 2\theta_{12} \sin^2 \alpha)^{\frac{1}{2}}, \quad (49)$$

depends on two parameters: $\sin^2 \alpha$ (α is the Majorana CP phase difference) and a common mass m_0 . From (49) and (11) we have the range

$$0.26 m_0 \leq |m_{ee}| \leq m_0 \quad (50)$$

Thus, in the case of the practically degenerate Majorana neutrino mass spectrum the scale of the effective Majorana mass $|m_{ee}|$ is determined by (unknown) common mass m_0 .

The mass m_0 can be determined from the data of the experiments on the measurement of the high-energy part of the β -decay spectrum of tritium and from cosmological data. In the tritium experiment KATRIN [39, 45], now at preparation, the sensitivity $m_0 \simeq 0.2 \text{ eV}$ is expected.

If in the future $0\nu\beta\beta$ -decay experiments it will be found that the value of the effective Majorana mass is significantly larger than $\sqrt{\Delta m_{32}^2} \simeq$

$4.5 \cdot 10^{-2} \text{eV}$ from $0\nu\beta\beta$ -decay data an information about the value of the common mass m_0 can be inferred:

$$|m_{ee}| \leq m_0 \leq \frac{|m_{ee}|}{\cos 2\theta_{12}} \quad (51)$$

Using the existing data (see (11)) we have

$$|m_{ee}| \leq m_0 \leq 3.8 |m_{ee}|$$

In conclusion we would like to emphasize that the measurement of the effective Majorana mass $|m_{ee}|$ could allow to obtain an important information on the pattern of the Majorana neutrino mass spectrum.

6 The problem of the nuclear matrix elements

Neutrinoless double β -decay is a second order in the Fermi constant G_F process with a virtual neutrino. For small neutrino masses (much smaller than the bounding energy of nucleons in nuclei) the matrix element of the $0\nu\beta\beta$ -decay is factorized in the form of a product of the effective Majorana mass $|m_{ee}|$, which depends on neutrino masses and elements U_{ei}^2 of the mixing matrix, and nuclear matrix element (NME), which is determined only by the strong interaction. The NME is a matrix element of the chronological product of the two CC hadronic currents and the neutrino propagator. It can not be connected with other observables. In the calculation of NME many intermediate states must be taken into account.

Two basic methods of the calculations of NME are used : quasiparticle random phase approximation (QRPA) and nuclear shell model (NSM). Many calculations of NME of different nuclei, based on these approximate approaches, exist in literature (see [26, 27, 29]). The results of different calculations of NME differ by about factor three or more. For example, if we assume that $|m_{ee}| = 5 \cdot 10^{-2} \text{eV}$ from different calculations of NME for the half-life of the $0\nu\beta\beta$ -decay of ^{76}Ge the values in the range

$$6.8 \cdot 10^{26} \text{y} \leq T_{1/2}^{0\nu}(^{76}\text{Ge}) \leq 70.8 \cdot 10^{26} \text{y}$$

can be obtained [29].

Recently a progress in the calculation of NME in the framework of QRPA have been achieved [46]. The nuclear matrix elements of $0\nu\beta\beta$ - decay of ^{76}Ge , ^{100}Mo , ^{130}Te and ^{136}Xe were calculated with the values of the parameter of the effective particle-particle interaction g_{pp} determined from the measured life-time of the $2\nu\beta\beta$ - decay. It was shown that the values of nuclear matrix elements of the $0\nu\beta\beta$ -decay of these nuclei are stable under the change of the nuclear potential and the number of single particle states used as a basis. Moreover the matrix elements calculated in different QRPA models are practically the same (differ not more than 10%.)

7 Possible test of the models of NME calculations

Taking into account all uncertainties connected with the calculations of the nuclear matrix elements of the $0\nu\beta\beta$ - decay, it will be very important to find a possibility to test NME calculations. We will discuss here a possible test, based on the factorization property of the matrix elements of the $0\nu\beta\beta$ -decay (see[47]).

The proposed test can be realized if neutrinoless double β -decay of *several nuclei* A_i, Z_i ($i=1,2,\dots$) is observed. Using a model M of the calculation of the nuclear matrix elements, from Eq.(20) the value of the parameter $|m_{ee}|_{A_i, Z_i}^2(M)$ can be determined. The model M is compatible with the data if the relations

$$|m_{ee}|_{A_1, Z_1}^2(M) \simeq |m_{ee}|_{A_2, Z_2}^2(M) = \dots \quad (52)$$

are satisfied.

From (20) it follows that for any nuclei (A, Z) the product

$$|m_{ee}|_{A, Z}^2(M) |M^{0\nu}(A, Z)|_M^2$$

does not depend on the model M . Thus, for two different models M_1 and M_2 we have

$$|m_{ee}|_{A, Z}^2(M_2) = |m_{ee}|_{A, Z}^2(M_1) \eta^{M_2; M_1}(A, Z), \quad (53)$$

where

$$\eta^{M_2; M_1}(A, Z) = \frac{|M^{0\nu}(A, Z)|_{M_1}^2}{|M^{0\nu}(A, Z)|_{M_2}^2}. \quad (54)$$

In the Table I we present the values of the coefficient $\eta(A, Z)$ for the case of the matrix elements calculated in [48] (NSM) and in [46] (the latest QPRA calculations).

If $\eta(A, Z)$ depends on (A, Z) and one model is compatible with the data the other model in principle can be excluded. However, as it is seen from the Table I from the observation of the $0\nu\beta\beta$ -decay of ^{136}Xe and ^{130}Te it will be difficult to distinguish models [48] and [46]: the difference between $\eta(^{136}\text{Xe})$ and $\eta(^{130}\text{Te})$ is about 20% . It will be more easier to distinguish models [48] and [46] if $0\nu\beta\beta$ -decay of ^{76}Ge and ^{130}Te is observed.

Taking into account the existence of many models of the calculations of the nuclear matrix elements of the $0\nu\beta\beta$ -decay we can conclude that the observation of neutrinoless double β -decay of three (or more) nuclei would be an important tool in the solution of the problem of NME.

Table I

The parameter $\eta^{NSM;QRPA}(A, Z)$, determined by Eq. (54), for nuclear matrix elements of the $0\nu\beta\beta$ -decay, calculated in Ref.[48] (NSM) and in Ref.[46] (QRPA)

Nucleus	$\eta^{NSM;QRPA}$
^{76}Ge	3.1
^{130}Te	2.1
^{136}Xe	2.5

8 Conclusion

After the discovery of neutrino masses and neutrino mixing the problem of *the nature of neutrinos with definite masses* ν_i is one of the most fundamental. The establishment of the nature of ν_i will have a profound impact on the understanding of the mechanism of the generation of the neutrino masses and mixing.

The most sensitive to the small Majorana neutrino masses process is neutrinoless double β -decay of even-even nuclei. From today's data to following bounds for the effective Majorana mass can be inferred

$$|m_{ee}| \leq (0.3 - 1.2) \text{ eV}$$

New experiments on the search for $0\nu\beta\beta$ -decay of ^{130}Te , ^{76}Ge , ^{136}Xe , ^{100}Mo and other nuclei are in preparation at present. In these experiments the

sensitivity

$$|m_{ee}| \simeq \text{a few } 10^{-2} \text{ eV}$$

is envisaged.

The data of neutrino oscillation experiments allow to predict ranges of possible values of the effective Majorana mass for different patterns of the neutrino mass spectra. In order to obtain information on the neutrino mass spectrum it is important not only to observe $0\nu\beta\beta$ -decay but also to *determine* the value of the effective Majorana mass $|m_{ee}|$.

From the measured half-life of $0\nu\beta\beta$ -decay only the product of the effective Majorana mass and nuclear matrix element can be obtained. Existing calculations of the nuclear matrix elements of the $0\nu\beta\beta$ -decay differ by about a factor of three or more. The improvement of the calculations of the nuclear matrix elements is a real theoretical challenge. We have discussed here a possible method which could allow to test models of calculation of the nuclear matrix elements of the $0\nu\beta\beta$ -decay. The method is based on the factorization property of the matrix element of $0\nu\beta\beta$ -decay and require observation of the $0\nu\beta\beta$ -decay of several nuclei.

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